Chapter Four: "If"

Introduction: Conditionals

1. More Logical Form

The formal methods developed in Chapter Three demonstrate the validity of any argument stated in the language of "and," "or," and "not". But some intuitively valid English arguments still slip through the net of those methods. The following argument, for example, is simple enough to strike us as clearly valid.

1. If Rex's team lost, then Rex is upset.
2. Rex's team lost.

∴ Rex is upset.

Testing this argument for validity formally involves (i) getting its form, via translation, then (ii) testing that form.

Since the first premise contains no conjunction, disjunction, or negation phrases, our current translation methods treat it as a subject matter sentence, and assign it a sentence letter, "P".

1. If Rex's team lost, then Rex is upset.	**P**
2. Rex's team lost.	

∴ 3. Rex is upset.

The second premise likewise contains no conjunction, disjunction, or negation phrases, and is also assigned a sentence letter. Since the second premise doesn't mean the same as the first, we give it a different sentence letter, "Q".

1. If Rex's team lost, then Rex is upset.	**P**
2. Rex's team lost.	**Q**

∴ 3. Rex is upset.

The conclusion also contains no Chapter Three form phrases, and so is treated as a subject matter sentence. Not meaning the same as either of the premises, it is assigned a new sentence letter, “R”.

1. If Rex’s team lost, then Rex is upset.	**P**
2. Rex’s team lost.	**Q**
∴ 3. Rex is upset.	**R**

But the logical form this translation yields looks terribly invalid. We see straightaway that attempting a deduction for the argument will be quite hopeless.

1. P	Premise
2. Q	Premise
	Get: R

Applying semantic methods confirms this suspicion: the truth table for the argument locates a counterexample in the second valuation. This argument form is invalid.

1	**2**	**∴**
P	**Q**	**R**
1	1	1
1	**1**	**0**
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

And of course a truth tree yields the same verdict: since a path remains open to the end, the argument form is (again) invalid

P	
Q	
	R

This is a somewhat paradoxical result: while the English argument clearly looks valid, our formal methods insist that it's invalid.

VALID	**INVALID**
1. If Rex's team lost, then Rex is upset. 2. Rex's team lost. ——————— ∴ 3. Rex is upset.	**P** **Q** —— ∴ **R**

Now if the English argument had been very complex, we might doubt our intuitive judgments here – knowing, as we do, how intuitions can be overwhelmed by complexity. But this argument is very simple, not the least bit mind-boggling. So the formal methods do indeed appear to be malfunctioning.

In searching for the culprit here, it is well to remember that our formal test of validity has two parts: getting the form (translation), and testing the form (deductions or semantic methods). Either, or both, could be the problem. So we could modify the translation procedure, giving the English argument a different formal counterpart; or alter the testing methods to stamp the current form as valid.

A moment's reflection makes clear that that second option is a terrible idea. For if we simply stipulate that the form

P
Q
———
∴ **R**

shall hereby qualify as valid, we'll wind up counting as valid all sorts of terrible arguments – such as the following.

Cats are mammals.
Pennsylvania is a U.S. state
————————————————
∴ The Washington Monument is made of glass.

Socrates was from Greece.
William of Ockham lived in the Middle Ages.
————————————————
∴ Benjamin Franklin walked on the moon.

Each of these arguments is translated into the logic form stated above, but each is obviously invalid. Solving our original problem this way only trades it in for a bigger problem.

Modifying the translation methods looks like a better alternative.

And there was already reason to suspect that the translation procedure was at fault, since it treats the three sentences of the valid argument as **completely unrelated**.

1. If Rex's team lost, then Rex is upset. **P**
2. Rex's team lost. **Q**
————————————
∴ 3. Rex is upset. **R**

In fact there are obvious overlaps. The second premise, for instance, already appeared as the **left half** of the **first premise**.

1. If **Rex's team lost**, then Rex is upset**.**
2. **Rex's team lost**.

∴ 3. Rex is upset.

Likewise the conclusion is the **right half** of the first premise.

1. If Rex's team lost, then **Rex is upset**.
2. Rex's team lost.

∴ 3. **Rex is upset**.

Our earlier formal translation papered over these connections between sentences. In particular, by translating the first premise as "P," it treated that sentence like a **logical atom**. But since the first premise has smaller sentences as parts, it's really a logical **molecule**. And logical molecules have bits of **logical form** connecting together their parts.

To isolate that logical form, we assign sentence letters to the parts of the first premise. Then a translation begins like so.

P: Rex's team lost
Q: Rex is upset

1. If P, then Q
2. P

∴ 3. Q

Assuming all the subject matter has been replaced by sentence letters, the remaining English phrase "**if… then**" is revealed as a bit of **logical form**.

And recognizing that, we see exactly what went wrong with the original translation: it **overlooked a piece of logical form**. The language of Chapter Three recognized "and," "or," and "not" (and their variations) as form; but it overlooked "if….then". We need to **expand the logical language**, to include this neglected bit of logical form.

2. Conditionals.

Just as we didn't rest content with labels such as "'and' sentence" and "'or' sentence," instead coining the jargon "conjunction" and "disjunction," here we settle on an official label for sentences of the "if… then" variety. Such a sentence is called a **conditional**.

Having this technical term handy will prove convenient later, when discussing the complications of English conditionals.

The sentence "*If Rex's team lost, then Rex is upset*" is a conditional of English. A corresponding formal conditional is then called for, to model this sentence in the formal language. For that purpose we introduce a new connective into the formal language: the "arrow".

$$\rightarrow$$

Using a single connective to translate a two-part phrase is familiar from the previous chapter, where "both… and" and "either… or" were likewise translated by a single connective (the wedge and vel, respectively).

Introduction of the arrow into the formal language is made official by adding a new construction rule for formal conditionals.

> 5. If ● and ▲ are formal sentences, then **(● → ▲)** is a formal sentence.

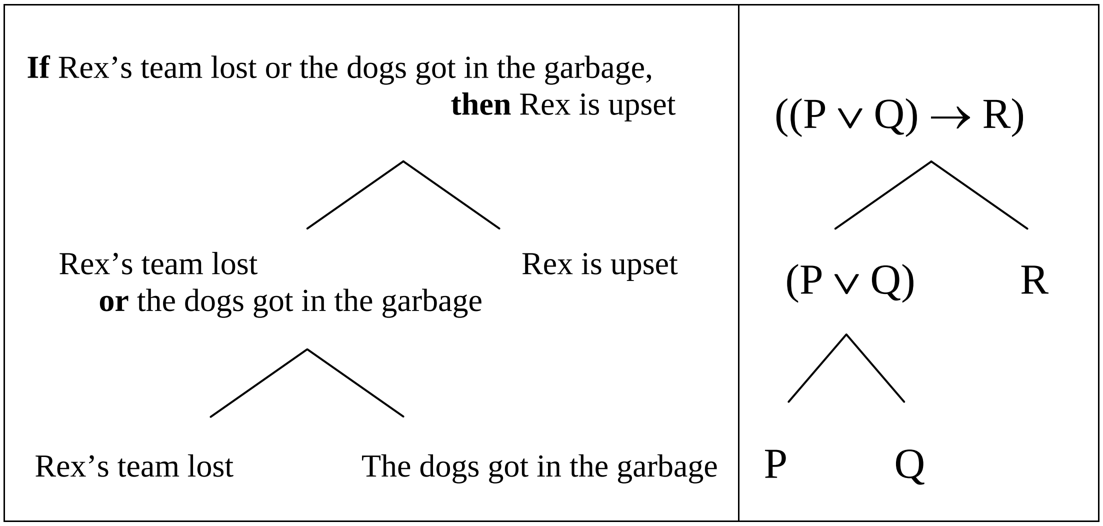
This addition allows formal translation of our earlier English conditional.

P: Rex's team lost
Q: Rex is upset

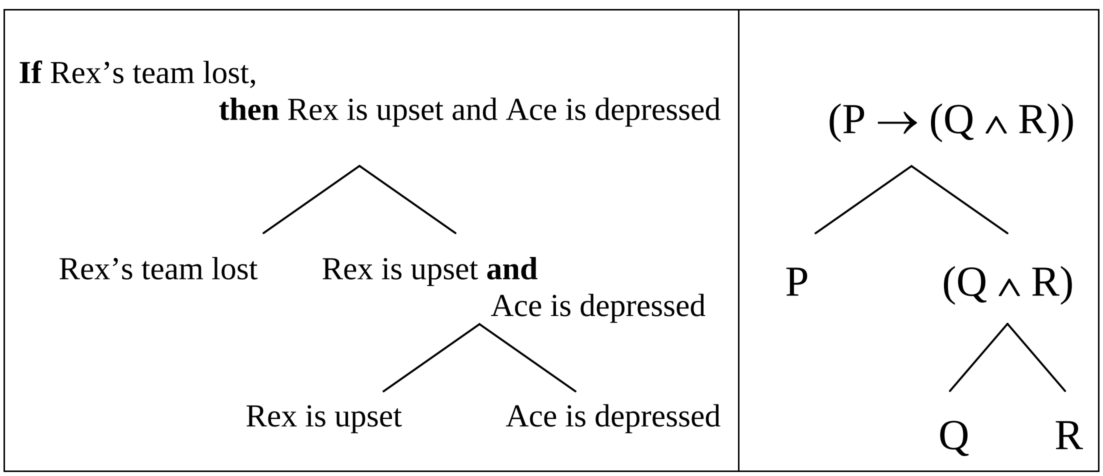
If Rex's team lost, then Rex is upset.

(P → Q)

But thanks to the ‘recycling’ (recursive) nature of the new construction rule, more complex English conditionals can be handled as well. So in both English and formal conditionals, the left part need not be an atomic sentence.



The right part can likewise be molecular.



In the wake of formal conjunctions and disjunctions, which likewise bring together a left and right part, this is familiar territory.

We turn next to translation. While the two complications translation brings are likewise familiar from the previous chapter, we find that with conditionals these complications spell trouble in a novel way.

Formal Language, Chapter Four

1. Sentence letters are formal sentences.

2. If ● is a formal sentence, then ~● is a formal sentence.

3. If ● and ▲ are formal sentences, then **(**● $\wedge$ ▲**)** is a formal sentence.

4. If ● and ▲ are formal sentences, then **(**● $\vee$ ▲**)** is a formal sentence.

5. If ● and ▲ are formal sentences, then **(**● $\rightarrow$ ▲**)** is a formal sentence.